Experimental evidence of frequency entrainment between coupled chaotic oscillations

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The nonlinear response of a chaotic system to a chaotic variation in a system parameter is investigated experimentally. Clear experimental evidence of frequency entrainment of the chaotic oscillations is observed. We show that analogous to the frequency locking between coupled periodic oscillations, this effect is generic for coupled chaotic systems. [S1063-651X(98)09903-6]

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Recently, on the basis of a numerical simulation, Stone has shown that a phase coherent strange attractor can be frequency entrained to small periodic driving pulses near the peak frequency in the broadband power spectra of the attractor [1]. Here by the term ''phase coherent attractor'' she means a strange chaotic attractor with the characteristic that its Fourier power spectrum has a sharp spectral peak, possibly together with its harmonics, in a continuous broadband spectral background. The appearance of a sharp spectral peak in the power spectrum indicates the existence of a predominant frequency in the chaotic oscillation [2]. Therefore, Stone's numerical result demonstrates that, analogous to the effect of the frequency locking of a driven periodic oscillations, there exists a similar effect in a driven chaotic oscillation. However, because for a chaotic attractor the phase is intrinsically dynamically unstable, the effect is that the average chaotic pulsation frequency is locked to the frequency of the driving periodic pulsations.

In the context of chaos synchronization, Rosenblum *et al.* [3] have also discovered a phenomenon of phase synchronization between weakly coupled self-sustained chaotic oscillators. In the phase synchronization regime they found in their numerical studies that the phases of the coupled chaotic oscillators are locked, while the amplitudes of them vary chaotically and are practically uncorrelated. As far as we understand, the so-called ''phase synchronization'' is in fact a state of frequency entrainment between the coupled chaotic attractors where their average chaotic pulsation frequencies are locked.

Although the possibility of frequency entrainment of chaotic attractors has been numerically demonstrated, to the best of our knowledge no clear experimental evidence of the effect in any physical chaotic systems has been reported so far. To explore the possibility and also to study the interaction between coupled chaotic systems, we have conducted an experimental investigation on the dynamics of a chaotic system subjected to a chaotic driving force in the form of a chaotic variation of a system parameter. Experimentally this is much easier to achieve than coherently adding to one of the system variables, as has been previously considered theoretically. For this reason it is also a case of greater technological significance. We have experimentally observed the frequency entrainment of the driven chaotic system to the chaotic driving signal. In this paper we present our experimental results. In particular, we show that the frequency entrainment effect is a generic behavior of coupled chaotic oscillations in the sense that it is insensitive to the coupled chaotic dynamics.

Our experiment is carried out on an optically pumped $^{15}NH_3$ single mode ring laser operating chaotically. Details about the laser and its configuration can be found in [4]. To study the effect of frequency entrainment, the pump intensity of the laser is chaotically modulated. This is experimentally done with the following procedure: Firstly we either record a chaotic intensity wave form from the laser with constant pump power or calculate a chaotic wave form with the Lorenz equations. We then store this chaotic wave form in the memory of an arbitrary function generator (AFG) (Hewlett Packard 33120A). Based on the stored chaotic wave form, the AFG produces an analog chaotic signal output whose chaotic wave form is the same as that of the stored signal, but its amplitude and average chaotic pulsation frequency can be continuously tuned. The chaotic output of the AFG is then used to amplitude modulate the rf driving signal of an acousto-optic modulator (AOM), and the undiffracted beam from the AOM is used as the $^{15}NH_3$ laser pump beam. Effectively the pump intensity obtained in this way consists of a chaotically varying pump intensity, which acts as the chaotic driving force in our experiment and a dc pump intensity, which ensures that even without the chaotic driving, the dynamics of the laser is chaotic. The chaotic intensity output of the $^{15}NH_3$ laser is detected with a Schottky-barrier diode. To compare the chaotic laser dynamics with those of the actual driving signal, the diffracted beam from the AOM is detected with a HgTe-CdTe detector. The electrical signals from both detectors are low-noise amplified and then displayed simultaneously on a Tektronix digital storage oscilloscope. A rf spectral analyzer is used to monitor the real-time Fourier power spectrum of the chaotic laser dynamics. Based on this information the frequency relationship between the chaotic laser dynamics and the driving chaotic signal is controlled.

The chaotic dynamics of the laser with constant pump have been intensively studied previously and are well known now [5,6]. Deterministic chaos such as period-doubling chaos, spiral chaos, and intermittency are typical chaotic dynamics of the laser. Under suitable conditions it was found that the chaotic dynamics of the laser can be well described by the single mode laser Lorenz-Haken equations [7] or extended Lorenz-Haken equations including the laser cavity

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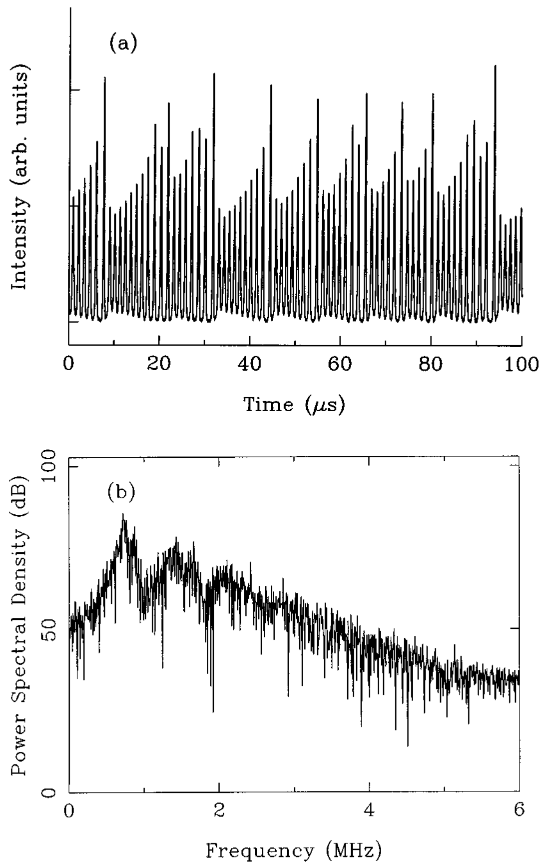

FIG. 1. A typical chaotic dynamics of the laser without chaotic driving. Pump intensity is 0.8 W/cm^2. Gas pressure is 19 μbar. Laser cavity tuning is close to the resonance. (a) Chaotic laser intensity evolution. (b) Fourier power spectrum calculated from the laser intensity evolution shown in (a).

detuning [8]. In our present experiment we have operated the laser in the parameter regime where its chaotic dynamics can be well understood with the extended laser Lorenz-Haken equations. For the sake of comparison, we show in Fig. 1 a typical experimentally observed chaotic dynamics of the laser without chaotic driving. Figure 1(a) is the chaotic evolution of the laser intensity. Figure 1(b) is the calculated Fourier power spectrum of the intensity evolution. As can be clearly identified from the Fourier spectrum, there is a spectral peak in the broadband spectrum. As a feature of the detuned laser Lorenz-Haken equations, when the laser cavity detuning is reduced to zero, the intensity dynamics of the laser moves from period-doubling chaos to Lorenz type spiral chaos [8]. Experimentally we observed that accompanying these dynamics changes, the width of the spectral peak broadens. The broadening of the spectral peak is a result of the increasing chaotic phase fluctuation associated with the Lorenz type spiral chaos, which is manifested as the variation in the time intervals between the successive chaotic laser intensity pulsations [1,2]. The position of the spectral peak represents the average pulsation frequency of the chaotic oscillation, and varies as experimental conditions such as the pump laser intensity, gas pressure, and the laser cavity detuning are changed. We have studied the chaotic dynamics of the laser with a chaotically varying pump by examining how its dynamics changes and the relationship of its dynamics to that of the driving signal. With a fixed driving signal

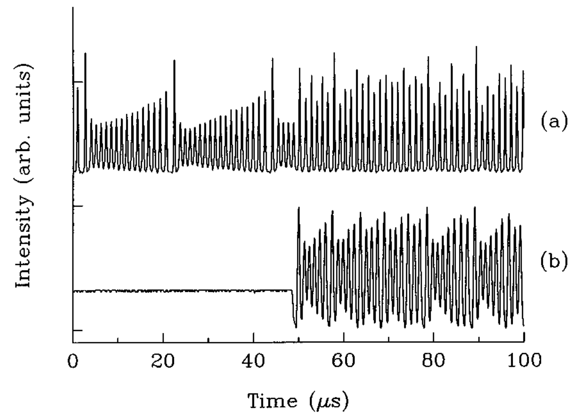

FIG. 2. Chaotic dynamics of the laser under frequency entrainment. Average pump intensity is 0.85 W/cm^2. Gas pressure is 19 μbar. Laser cavity tuning is almost in resonance. The peak-peak modulation strength is 20% of the mean pump intensity. (a) Chaotic laser intensity evolution. (b) Evolution of the driving chaotic signal.

strength, we experimentally observed that the separation between average pulsation frequency of the driving signal and that of the chaotic laser intensity plays an important role in the chaotic dynamics of the laser. When the separation between the average frequencies is large, under a chaotic pump the dynamics of the laser becomes more complicated than its original single mode dynamics. In this case the chaotic intensity evolution of the laser exhibits no obvious relation to the driving signal. However, when the average frequency separation is small, within a certain range that solely depends on the driving signal strength (with a peak-peak chaotic modulation of 30% of the average pump intensity, we observed a locking range of about 0.15 MHz), the average pulsation frequency of the laser locks to that of the driving signal and the chaotic intensity pulsations of the laser become frequency entrained to that of the driving signal.

To demonstrate this frequency entrainment effect, we show in Fig. 2 a typically observed experimental result. Figure 2(a) is the time evolution of the chaotic laser intensity, Fig. 2(b) is the chaotic driving signal measured by inverting the intensity of the power diffracted by the AOM. We have made a part of the modulation signal a dc level so as to compare the laser dynamics with and without chaotic driving. The original chaotic wave form used to modulate the rf driving signal of the AOM is a segment of prerecorded chaotic intensity evolution of the laser without chaotic driving, and the dc level is selected such that it is the mean level of the chaotic signal. However, due to the bandwidth limitation of the AOM, the real chaotic wave form of the driving shown in Fig. 2(b) is significantly distorted from the original wave form. This drawback shows on the other hand that the observed effect is not dependent on the detailed driving signal dynamics. Figure 2(a) shows that without the chaotic driving the intensity dynamics of the laser is a Lorenz spiral chaos. When the chaotic driving is switched on, the dynamics of the laser deviates from its original form and normally after a transient that depends on the driving strength and the average frequency separation, the chaotic intensity pulsations of the laser become ''synchronized'' to the chaotic driving pulses, despite the fact that both the driving signal and the intrinsic chaotic laser dynamics possess strong chaotic phase fluctuations. Once the average frequencies are locked, slightly changing the experimental conditions such as the pump frequency detuning does not change the average pulsation frequency. When the driving signal is switched off, the chaotic dynamics of the laser returns to its original Lorenz spiral chaos.

We have examined in detail the relationship between the chaotic laser pulsations and the driving signal. While without the chaotic driving, the successive peak intensity pulse return map of the laser shows a clear cusp form of structure normally found for spiral chaos [5], under the frequency entrainment shown in Fig. 2, this cusp structure smears out. By measuring the time delay as the time interval between the peak of the laser intensity pulse to its corresponding driving pulse, we found that the delay is not fixed but the higher the driving pulse, the smaller the time delay. Plotting the delay time return map does not show any relationship between them either. Obviously, in this state the amplitudes of the chaotic laser intensity pulsations are independent of those of the driving signal.

To quantitatively demonstrate that the average chaotic pulsation frequency of the laser shown in Fig. 2 is locked to that of the driving signal, we have calculated the instantaneous phase evolution associated with the chaotic driving signal and the chaotic laser intensity evolution based on analytic signal theory [9]. The average overall slope of the chaotic phase evolution gives the average chaotic pulsation frequency. As shown in Fig. 3 these two phase evolutions have the same average slope. As also evidenced by Fig. 3, the instantaneous phases vary independently about the average phase slope. Since the deviation of the instantaneous phase from the average phase slope is a result of the chaotic phase modulation due to the chaotic dynamics, this experimental result shows that under the frequency entrainment, the phase dynamics of the driven oscillation can still be chaotic and not related to the driving signal.

We have also experimentally studied the possibility of frequency entraining the chaotic dynamics of the laser with other chaotic driving signals; e.g., we have used a period-doubling chaos wave form as the driving signal to frequency entrain the Lorenz-like spiral chaos of the laser and vice versa. We have also used chaotic wave forms calculated from the Lorenz equations as the driving signal and repeated the experiment. In all cases we have observed the same effect. Our experimental results suggest that this effect is a generic feature of coupled chaotic oscillations and it is insensitive to the coupled chaotic dynamics.

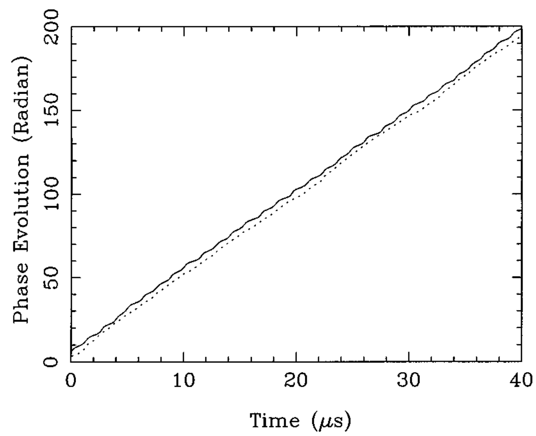

FIG. 3. Chaotic phase evolution of the laser intensity evolution and the driving signal. Solid line: phase evolution calculated from the laser intensity evolution. Dotted line: phase evolution calculated from the driving signal.

In conclusion, we have experimentally observed the frequency entrainment of a chaotic oscillation to a chaotic driving signal. Our experimental results show that although for a chaotic oscillation there is no unique natural frequency, under coupling it can exhibit an effect similar to the frequency entrainment of a coupled periodic oscillator. However, in the chaotic case it is the average chaotic pulsation frequencies that are locked together. This experimental result suggests that in a chaotic oscillation, the average chaotic pulsation frequency plays the same role as that of the natural frequency of a periodic oscillation. Because under the frequency entrainment only the average chaotic pulsation frequency is constrained and there is no restriction on the phase and amplitude variations, and for a chaotic oscillation its phase and amplitude are intrinsically dynamically unstable, these result in that, under the frequency entrainment, the dynamics of the coupled chaotic systems are still chaotic and irrelevant to each other. Finally, our experimental results with several forms of chaos demonstrate that the frequency entrainment effect is a generic effect of coupled chaotic oscillations, and it is independent of the particular coupled chaotic dynamics.

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